Risk Assessment in Geotechnical Engineering

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Risk Assessment and Mitigation in Geotechnical Practice

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University of California at Santa Cruz

THE BANANA SLUG

"……in earthwork engineering the designer has to deal with bodies of earth with a complex structure and **the properties of the material may vary from point to point.**"

K. Terzaghi

Prefce to the Inaugural Edition of *Géotechnique* (1948)

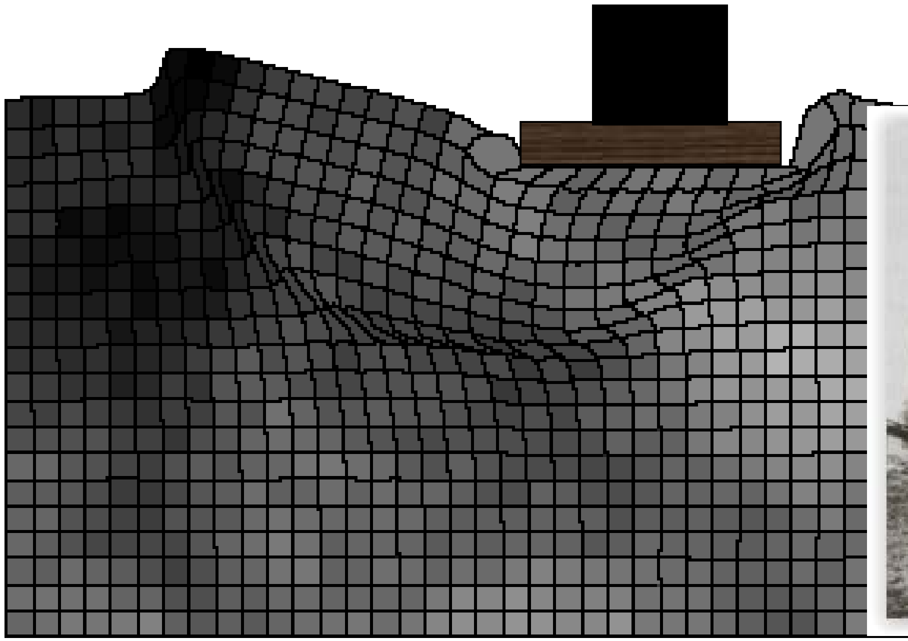
"Two specimens of soil taken at points a few feet apart, even if from a soil stratum which would be described as relatively homogeneous, **may have properties differing many fold.**"

Donald W. Taylor

Introduction to *Fundamentals of Soil Mechanics*
Wiley, (1948)

It is only relatively recently however, that methodologies such as the Random Finite Element Method (RFEM) have been developed to explicitly model the variability discussed by Terzaghi and Taylor.

Bearing Capacity



Bearing failure of a silo in Manitoba, Canada (1913)

Outline

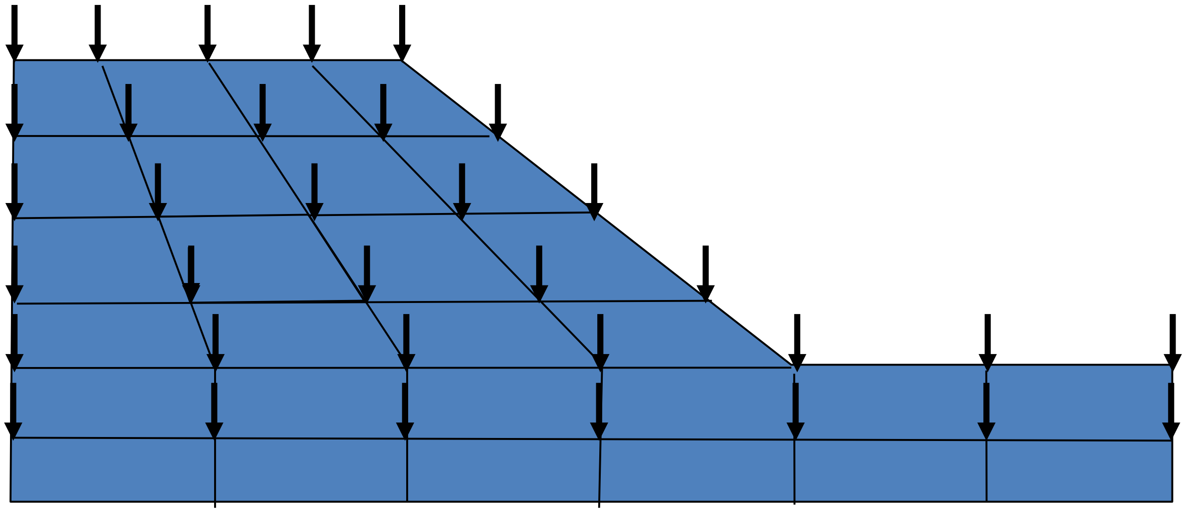
1. **Slope Stability Analysis by Finite Elements**
   - **“Seeking out failure”**
   - **Variable soils**

2. **Risk Assessment in Geotechnical Engineering**
   - **Three levels of probabilistic analysis**
     - ➢ **Event Trees**
     - ➢ **First Order Methods**
     - ➢ **Monte Carlo**
   - **Modeling spatial variability.**
     The Random Finite Element Method (RFEM)

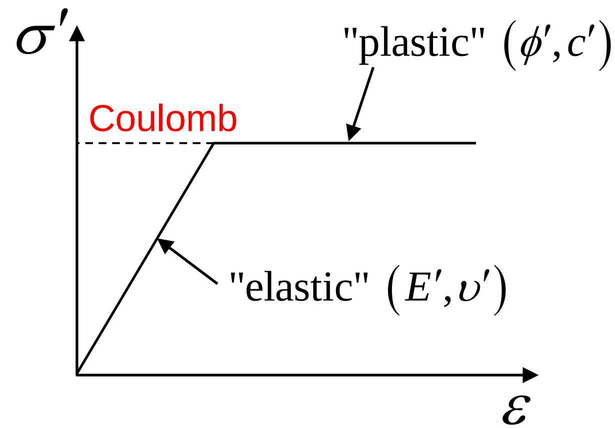
3. **Concluding Remarks**

1. Slope Stability Analysis by Finite Elements

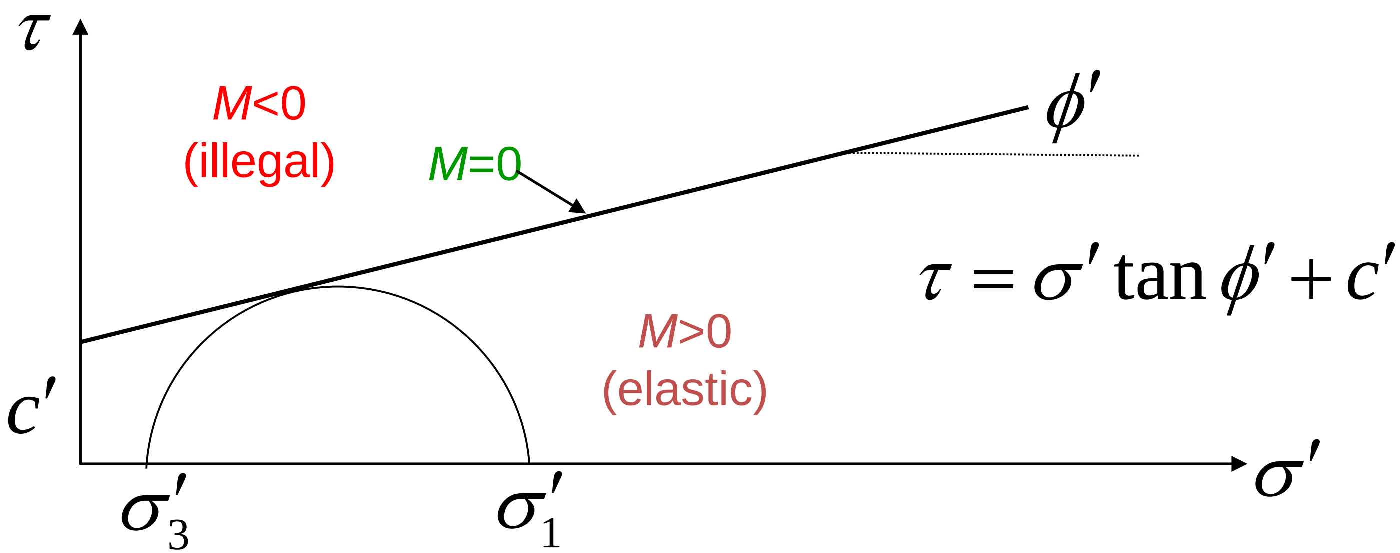
- Gravity loads are applied to the mesh.

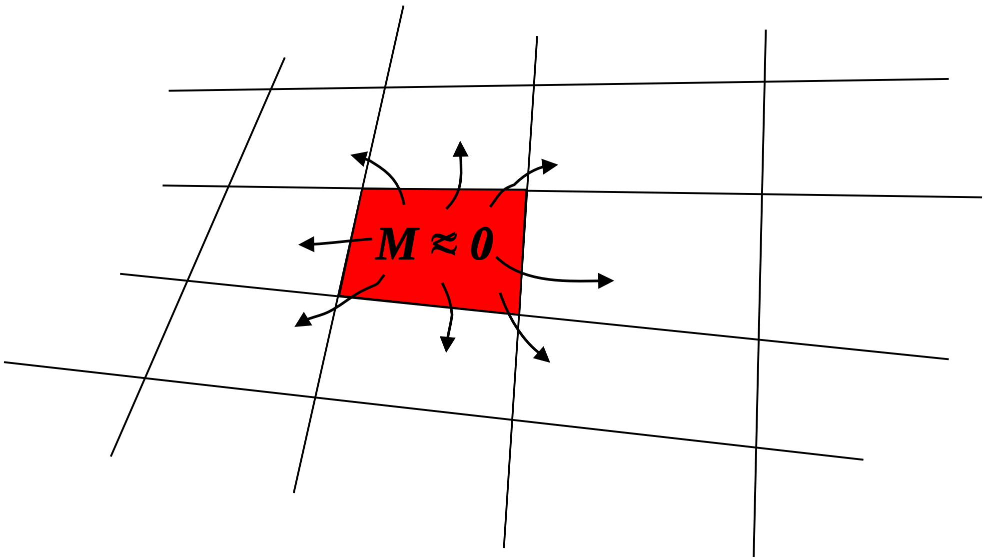


soil is given simple elastic-perfectly plastic stress-strain model



- Compute elastic stresses and check for elements violating Coulomb



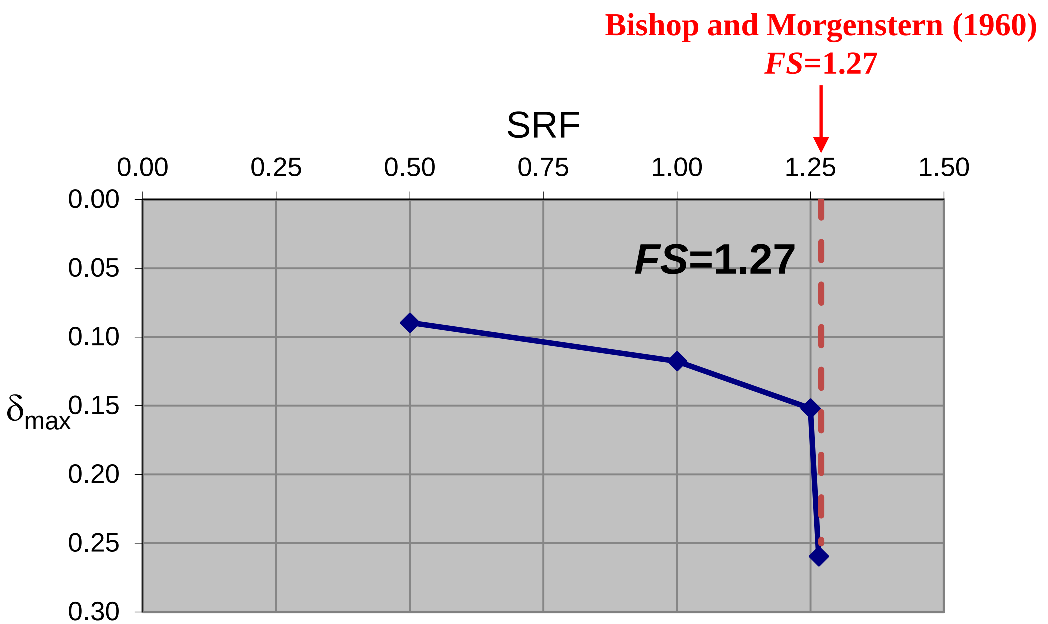


- Element with elastic stresses violating Coulomb ($M < 0$)
- Stress redistribution while maintaining global equilibrium

- <u>Strength reduction to failure</u>

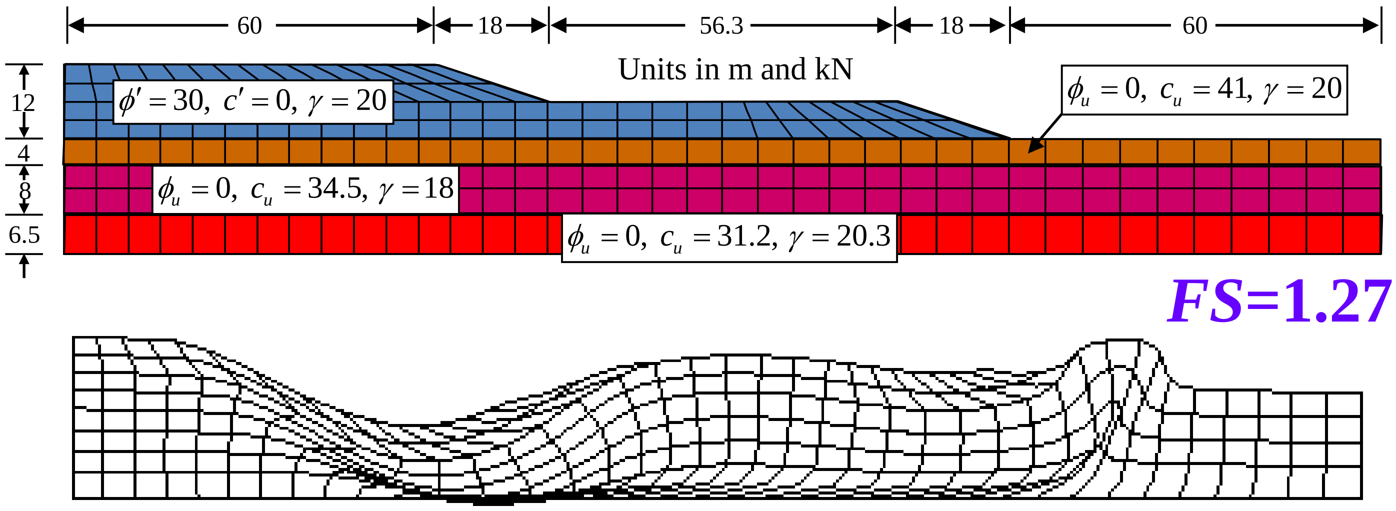
$$c'_f = \frac{c'}{SRF} \quad \phi'_f = \arctan\left(\frac{\tan\phi'}{SRF}\right)$$

At failure $FS \approx SRF$



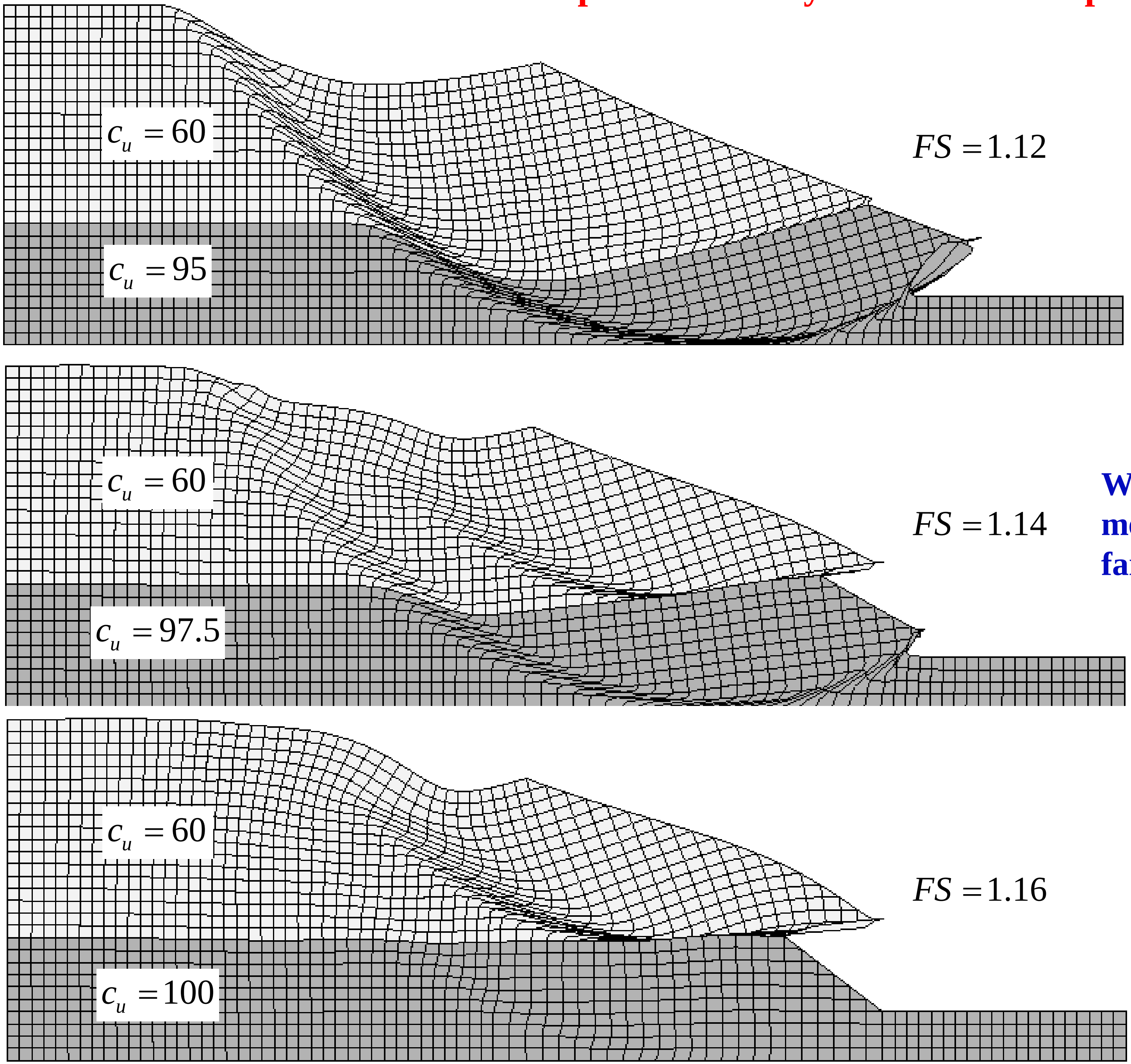
"Seeking out failure"

James Bay Dike using Finite Elements



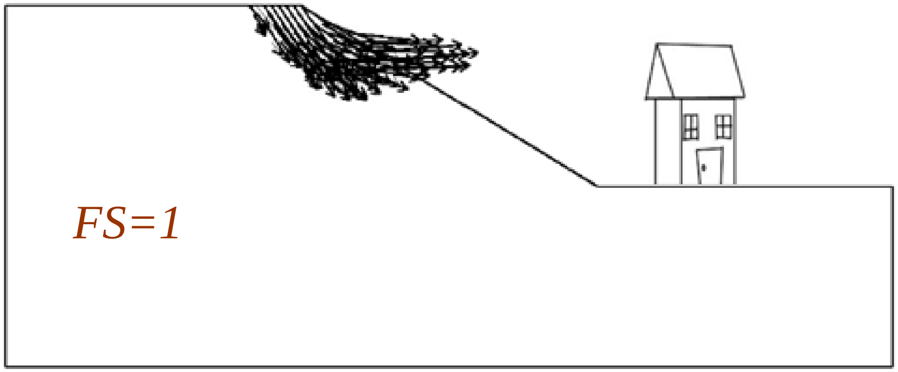
- Failure mechanism **"seeks out"** the path of least resistance.
- Slope fails **"naturally"** through zones where the shear strength is unable to resist the shear stresses.

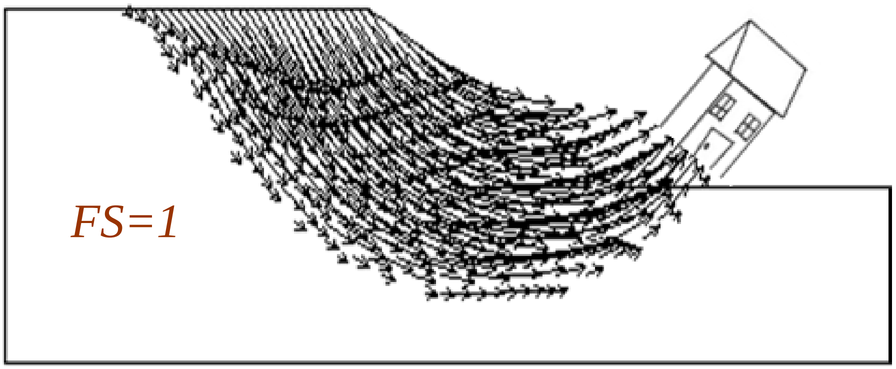
Another example with a 2-layer undrained slope.



Would a limit equilibrium method find both these failure mechanisms?

2) Risk Assessment in Geotechnical Engineering





Two slopes with the same factor of safety

WHAT ABOUT THE CONSEQUENCES OF FAILURE?

Definition of RISK

Probability of Failure
weighted by the
Consequences of Failure

What is acceptable risk?

Baecher (1982)

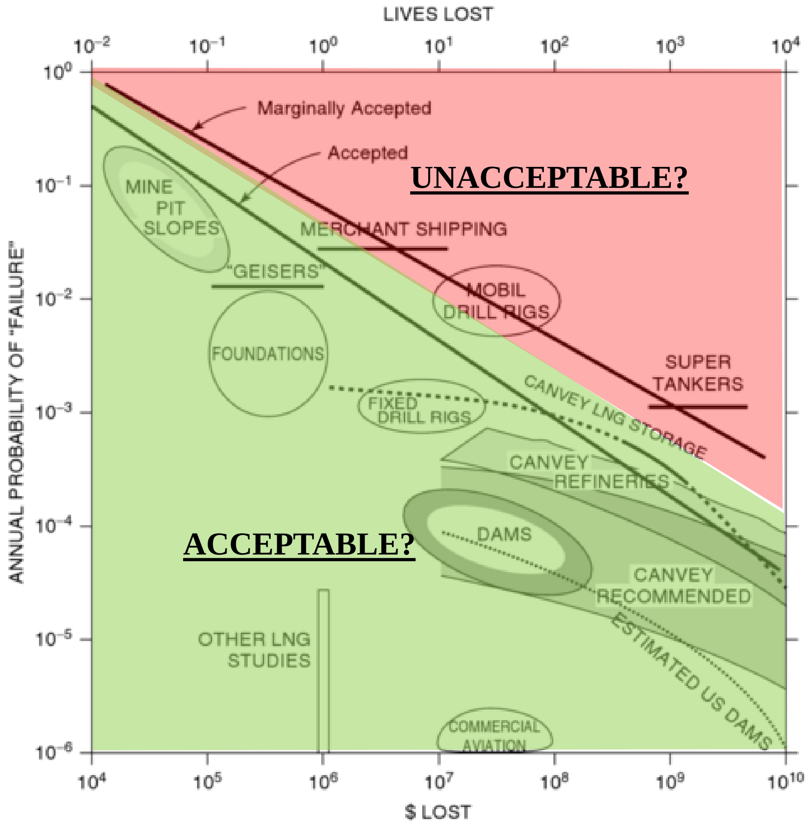


Figure 5.7 F-N chart showing average annual risks posed by a variety of traditional civil facilities and other large structures or projects (Baecher 1982b).

A Risk Assessment study starts with a Probabilistic Analysis

Goal of a probabilistic geotechnical analysis…..?

To estimate the "Probability of failure (p_f)" as an alternative, or complement to, the traditional "Factor of Safety (*FS*)"

Alternatives might be the

"Probability of inadequate performance"

"Probability of design failure"

Some investigators prefer a more optimistic terminology.....e.g.

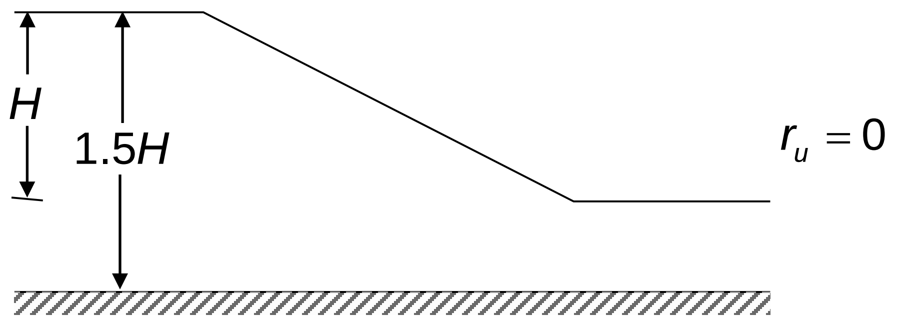
"reliability"

"reliability (index)"

.....so what, if any, is the relationship between p_f and *FS* ??

CONSIDER TWO EXAMPLES OF SLOPE STABILITY

Find the factor of safety of a 2H:1V slope shown:



Example 1

$$\phi' = 23^\circ$$

$$\frac{c'}{\gamma H} = 0.048$$

$FS = 1.5$

Solution from charts, e.g. Michalowski (2002),

Example 2

$$\phi' = 32^\circ$$

$$\frac{c'}{\gamma H} = 0.048$$

$FS = 2.0$

….so the slope in Example 2 is "safer"…..?

Following a probabilistic analysis we may get more information on the statistical distribution of the Factor of Safety in these Examples.

Suppose such an analysis reveals that:

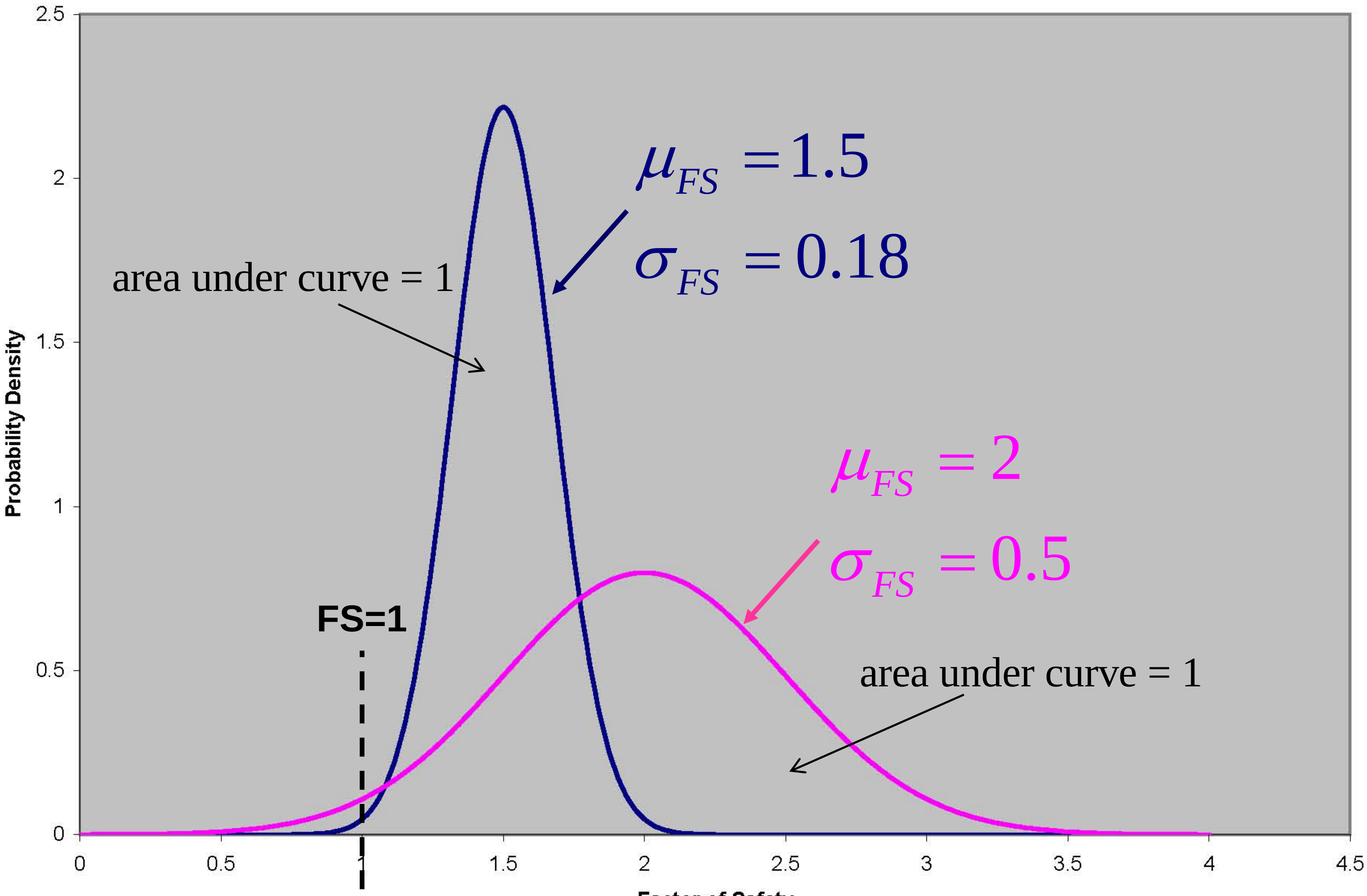
for Example 1:

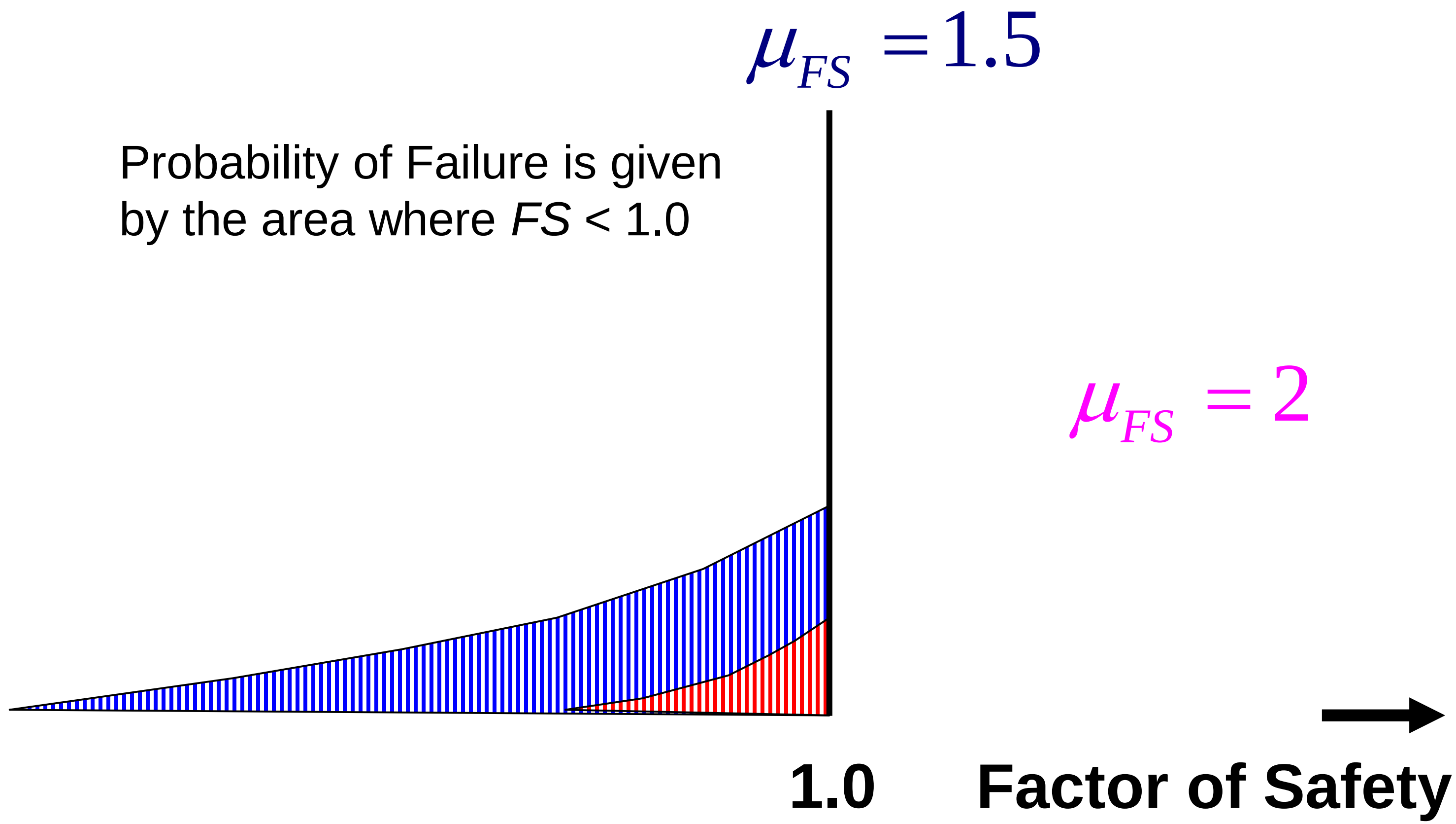
$$\mu_{FS} = 1.5, \ \ \sigma_{FS} = 0.18$$

and for Example 2:

$$\mu_{FS} = 2.0, \ \ \sigma_{FS} = 0.5$$

Consider once more, the two slopes from a probabilistic standpoint



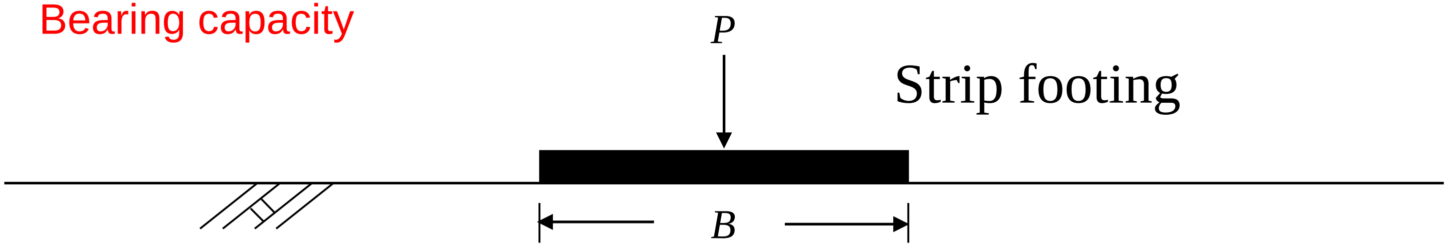


The "safer" slope has a higher "probability of failure"!

As tempting as it is....direct comparison between the Factor of Safety and the Probability of Failure should be done with great care.

Geotechnical Analysis: The Traditional Approach

Bearing capacity



ϕ' → , c' →

$$q_{ult} = c'N_c + qN_q + \frac{\gamma B}{2} N_\gamma$$

e.g. Terzaghi's bearing capacity equation

↓

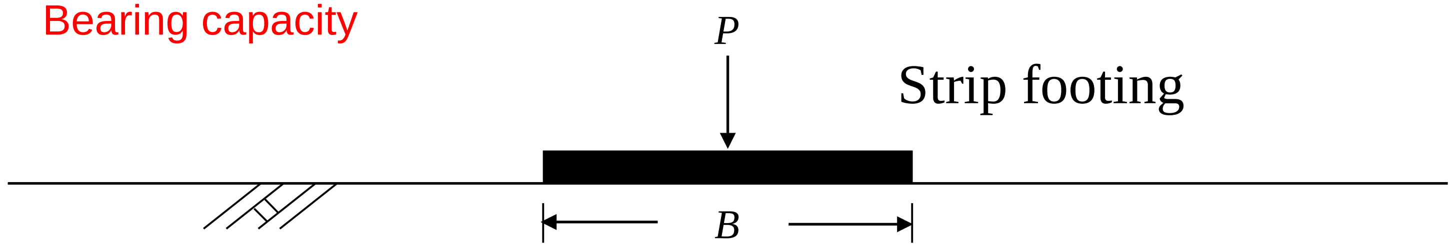
$$q_{ult}$$

↓

$$q_{all} = \frac{q_{ult}}{FS}$$

Geotechnical Analysis: The Probabilistic Approach

Bearing capacity



$\mu_{\tan\phi'}$ $\sigma_{\tan\phi'}$ $\sigma_{c'}$ $\mu_{c'}$

spatial correlation

cross correlation

$$q_{ult} = c'N_c + qN_q + \frac{\gamma B}{2} N_\gamma$$

Terzaghi's bearing capacity equation

$\mu_{q_{ult}}$ $\sigma_{q_{ult}}$

probability tables

$$p_f = \mathrm{P}\left[q_{ult} < q_{all}\right]$$

Fundamental Question

How does variability of input affect variability of output?

THREE LEVEL OF PROBABILISTIC ANALYSIS

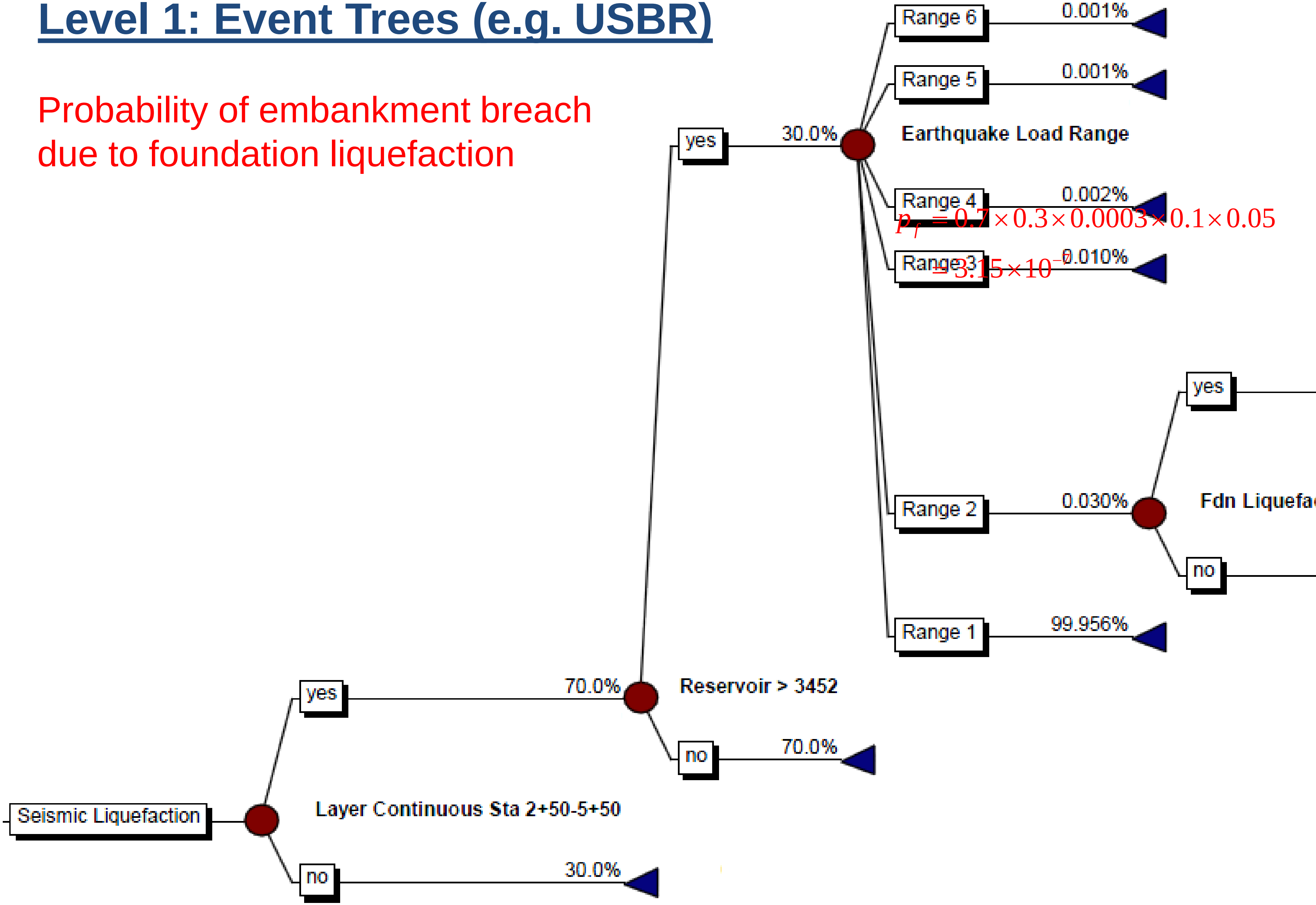
1. Expert Panel
   - Event Trees

2. First Order Methods
   - First Order Reliability Methods (FORM)

3. Monte-Carlo
   - Single random variable approach (SRV)
   - Random Finite Element Method (RFEM)

Level 1: Event Trees (e.g. USBR)

Probability of embankment breach due to foundation liquefaction

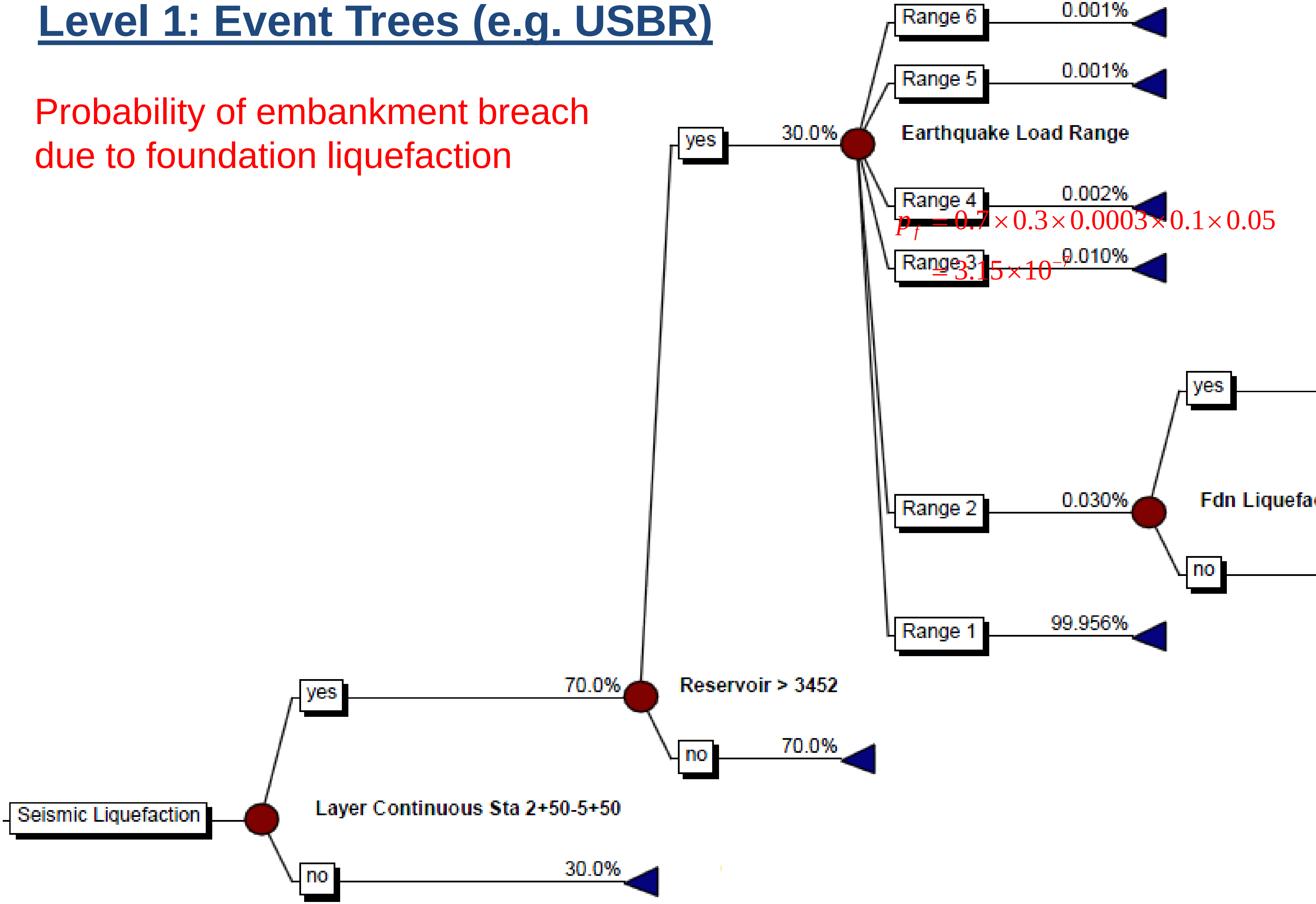


$$p_f = 0.7 \times 0.3 \times 0.0003 \times 0.1 \times 0.05$$
$$= 3.15 \times 10^{-7}$$

Level 1: Event Trees (e.g. USBR)

Probability of embankment breach due to foundation liquefaction

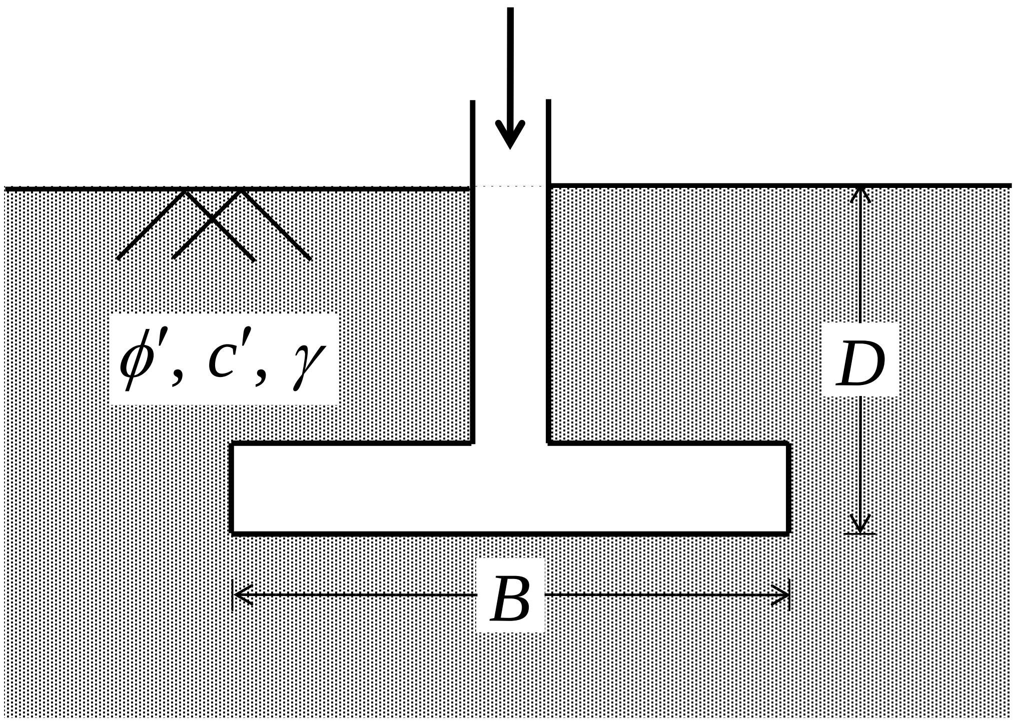
$$p_f = 0.7 \times 0.3 \times 0.0003 \times 0.1 \times 0.05 = 3.15 \times 10^{-7}$$



Level 2: First Order Reliability Method (FORM)

Probability of bearing capacity failure

$Q_{all} = 1200$



Square footing

$D = 1$

$B = 2$

Units in kN and m

$\gamma = 18$

$$q_{all} = \frac{Q_{all}}{2^2} = 300$$

$$p_f = P\left[q_{ult} < 300\right]$$

Random and correlated $\tan\phi'$ and c'

Variable	Mean	St Dev	Dist type
c'	4	1	Normal
$\tan\phi'$	0.577 (30^0)	0.086	Normal

$\rho = -0.3$

$$FS = \frac{q_{ult}}{q_{all}} = \frac{1071}{300}$$ =3.6 (based on mean values)

Level 2: First Order Reliability Method (FORM)

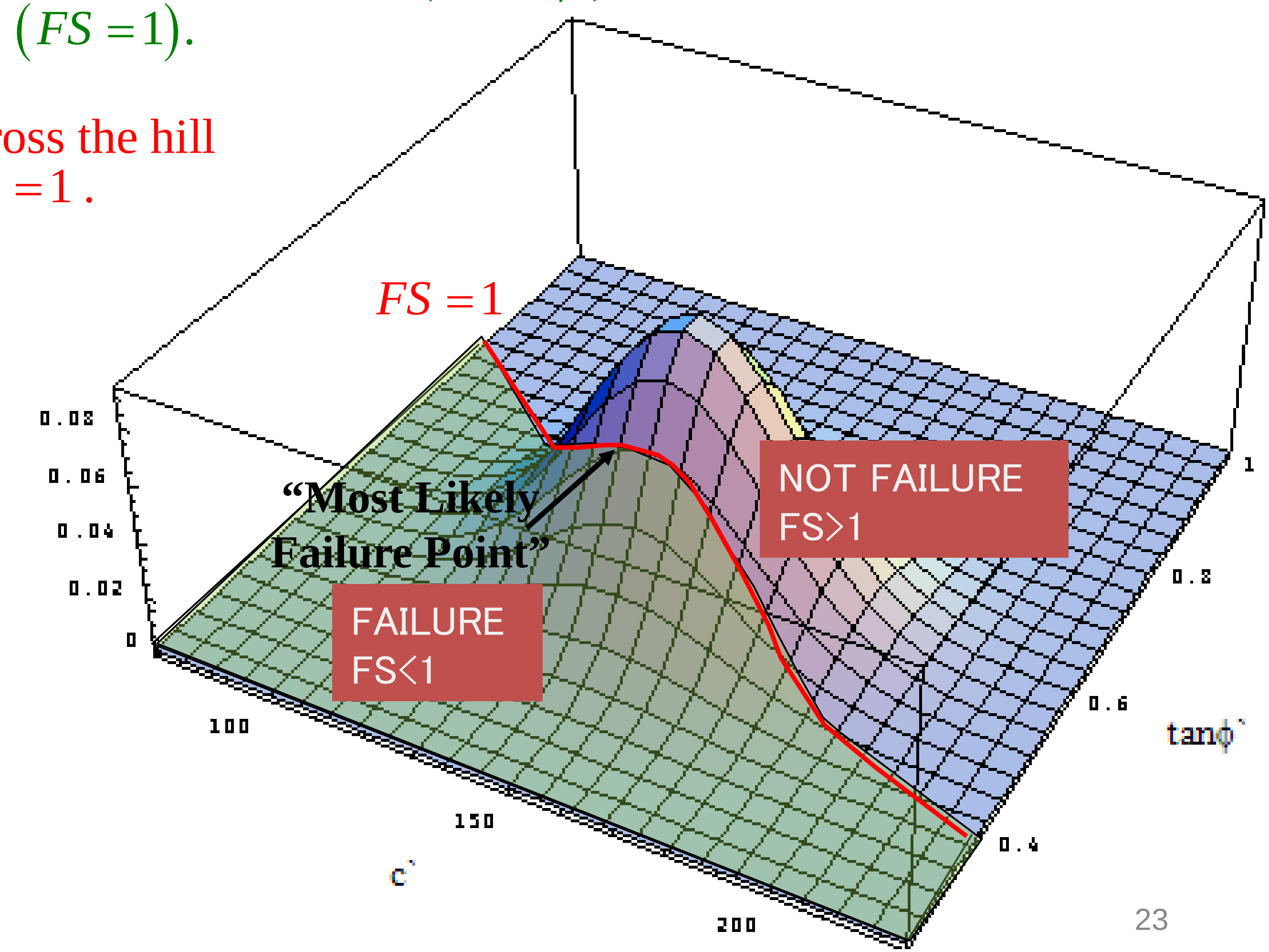
Consider a joint probabilty density function of c' and $\tan\phi'$ that might be used in a geotechnical stabilty problems of bearing capacity or slope stability.

There is an infinite number of combinations of $(c', \tan\phi')$ that might result in failure $\left(FS = 1\right)$.

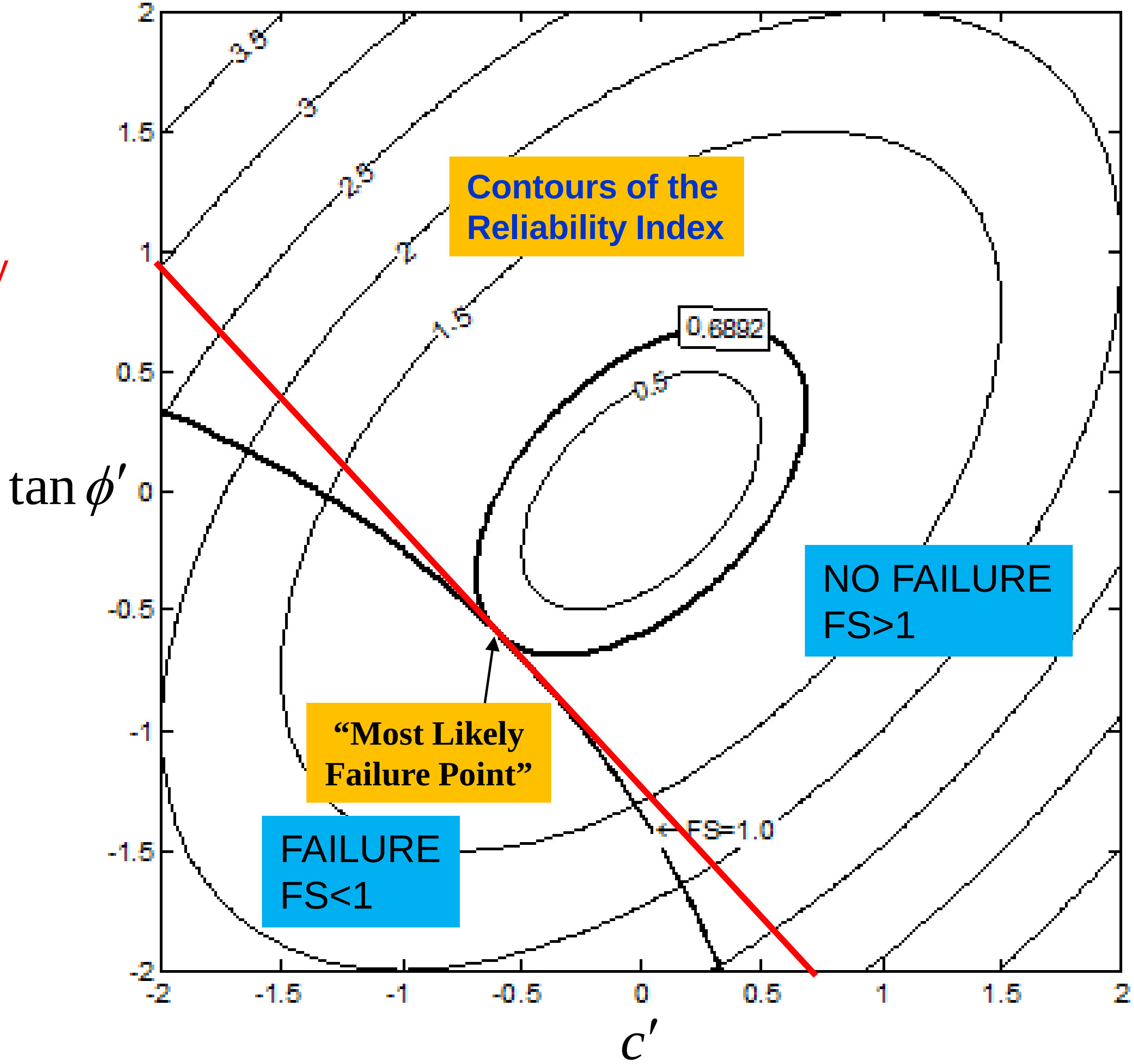
A vertical wall cutting across the hill represents the locus of $FS = 1$.

FORM will find the *most likely* values of c' and $\tan\phi'$ to cause failure.
i.e. the values *closest* to the top of the hill.

The probability of failure is the *volume* of the hill on the failure side of the $FS = 1$ line



First
Order
Reliability
Method

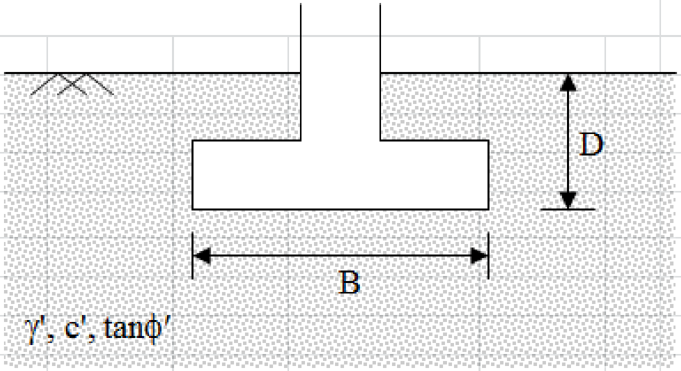


FORM computes p_f as the volume under the hill on the failure side of the <u>straight line</u>

First Order Reliability Method (FORM)

Example - Bearing Capacity of a Square Foundation

Data → Solver → Solve



Deterministic Variables

D	1	**Dimensions**
B	2	**m and kN**
γ	18	
q_{all}	300	

Probabilistic Variables

Variable	Mean	SD	Value	Reduced
c'	4	1	4.000	0.000
tanϕ'	0.577	0.086	0.577	0.000

Correlation

c'	tanϕ'
1	-0.3
-0.3	1

0.000	0.000	0.000
		0.000

q	18.00	F_{cs}	1.61
a_θ	6.13	F_{cd}	1.20
N_q	18.37	F_{qs}	1.58
N_c	30.10	F_{qd}	1.14
N_γ	22.35	$F_{\gamma s}$	0.60
q_{ult}	1070.8	$F_{\gamma d}$	1.00

Inverse of Correlation

1.099	0.330
0.330	1.099

$$\beta = \min_{g=0} \sqrt{\left[\frac{x_i - \mu_i}{\sigma_i}\right]^T [C]^{-1} \left[\frac{x_i - \mu_i}{\sigma_i}\right]}$$

Limit State Function, M	Reliability Index, β	Probability of Failure
2.569	0.000	50.00%

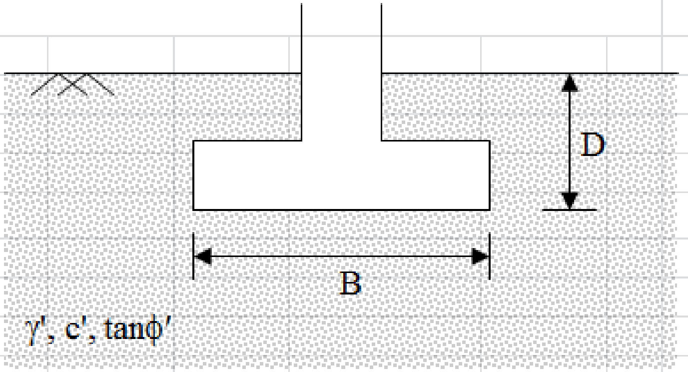
Limit State Function: $M = \dfrac{q_{ult}}{q_{all}} - 1$

First Order Reliability Method (FORM)

Example - Bearing Capacity of a Square Foundation

Deterministic Variables

D	1	**Dimensions**
B	2	**m and kN**
γ	18	
q_{all}	300	



Probabilistic Variables

Variable	Mean	SD	Value	Reduced
c'	4	1	4.402	0.402
tanϕ'	0.577	0.086	0.334	-2.825

Correlation

	c'	tanϕ'
c'	1	-0.3
tanϕ'	-0.3	1

q	18.00	F_{cs}	1.41
a_θ	2.86	F_{cd}	1.20
N_q	5.50	F_{qs}	1.33
N_c	13.49	F_{qd}	1.16
N_γ	4.34	$F_{\gamma s}$	0.60
q_{ult}	300.0	$F_{\gamma d}$	1.00

0.402	-2.825	0.402
		-2.825

Inverse of Correlation

1.099	0.330
0.330	1.099

$$\beta = \min_{g=0} \sqrt{\left[\frac{x_i - \mu_i}{\sigma_i}\right]^T [C]^{-1} \left[\frac{x_i - \mu_i}{\sigma_i}\right]}$$

Limit State Function, M

0.000

Reliability Index, β

2.864

Probability of Failure

0.21%

Limit State Function: $M = \dfrac{q_{ult}}{q_{all}} - 1$

Level 3: Monte-Carlo (Single Random Variable)

	A	B	C	D	E	F	G	H
1			**Generate Random Number with Correlation**					
2	c'		Simulation	X	Y	c'	tanϕ'	Correl.
3	Mean	SD	1	-0.0875	0.0506	3.9125	0.5834	-0.0006
4	4	1	2	-1.0061	-2.1688	2.9939	0.4250	0.1529
5			3	-0.0408	0.1292	3.9592	0.5887	-0.0005
6	**tanϕ'**		4	0.159				
7	Mean	SD						
8	0.577	0.086						
9							0.6526	-0.0789
10	**Correlation**						0.5171	0.0326
11	**c' - tanϕ'**						0.6077	0.0119
12	-0.3		10	-1.4207	-0.4429	2.5793	0.5773	-0.0005
13			11	1.1090	0.6429	5.1090	0.6011	0.0268
14			12	1.9514	-0.3754	5.9514	0.4959	-0.1583
15			13	0.3187	-0.0390	4.3187	0.5656	-0.0036

Font

f_x =NORMI

C	D	E

$\tan\phi' = \mu_{\tan\phi'}$

$(\mu_{c'} - c')(\mu_{\tan\phi'} - \tan\phi')$

etc.

99996			99994	0.5783	-0.7350	4.5783	0.5018	-0.0435
99997			99995	1.2395	1.0989	5.2395	0.6352	0.0721
99998			99996	0.9341	-0.2687	4.9341	0.5309	-0.0431
99999			99997	0.6949	0.7553	4.6949	0.6210	0.0306
100000			99998	0.3911	1.9083	4.3911	0.7235	0.0573
100001			99999	-1.1486	-0.2408	2.8514	0.5869	-0.0113
100002			100000	1.2028	1.0816	5.2028	0.6347	0.0694
100003			**Mean**	-0.0013	0.0002	3.9987	0.5771	-0.2992
100004			**SD**	1.0004	1.0050	1.0004	0.0864	
100005								

Chec $\mu_{c'}$ and

Check $\mu_{\tan\phi'}$ and $\sigma_{\tan\phi'}$

Check $\rho_{c',\tan\phi'}$

Compute bearing capacity of each Monte-Carlo simulation

1	Bearing Capacity Problem														
2	Simulation	c'	tanϕ'	ϕ'	N_q	N_c	N_γ	F_{cs}	F_{cd}	F_{qs}	F_{qd}	$F_{\gamma s}$	$F_{\gamma d}$	q_{ult}	# Fail
3	1	5.8942	0.4955	26.36	12.32	22.84	13.20	1.54	1.2	1.50	1.15	0.6	1	773.7	0
4	2	5.4478	0.5343	28.12	14.91	26.03	17.00	1.57	1.2	1.53	1.15	0.6	1	924.5	0
5	3	4.1769	0.5187	27.42	13.81	24.69	15.36	1.56	1.2	1.52	1.15	0.6	1	793.4	0
6	4	4.0859	0.6747	34.01	29.46	42.19	41.10	1.70	1.2	1.67	1.13	0.6	1	1799.7	0
7	5	4.5458	0.4865	25.95	11.78	22.17	12.44	1.53	1.2	1.49	1.15	0.6	1	683.4	0
8	6	5.1030	0.5121	27.12	13.36	24.15	14.71	1.55	1.2	1.51	1.15	0.6	1	807.5	0
9	7	2.4278	0.7218	35.82	36.91	49.75	54.73	1.74	1.2	1.72	1.12	0.6	1	2129.6	0
10	8	4.8523	0.3083	17.13	4.83	12.44	3.60	1.39	1.2	1.31	1.15	0.6	1	270.7	1
11	9	2.9899	0.7131	35.49	35.41	48.25	51.93	1.73	1.2	1.71	1.13	0.6	1	2089.8	0

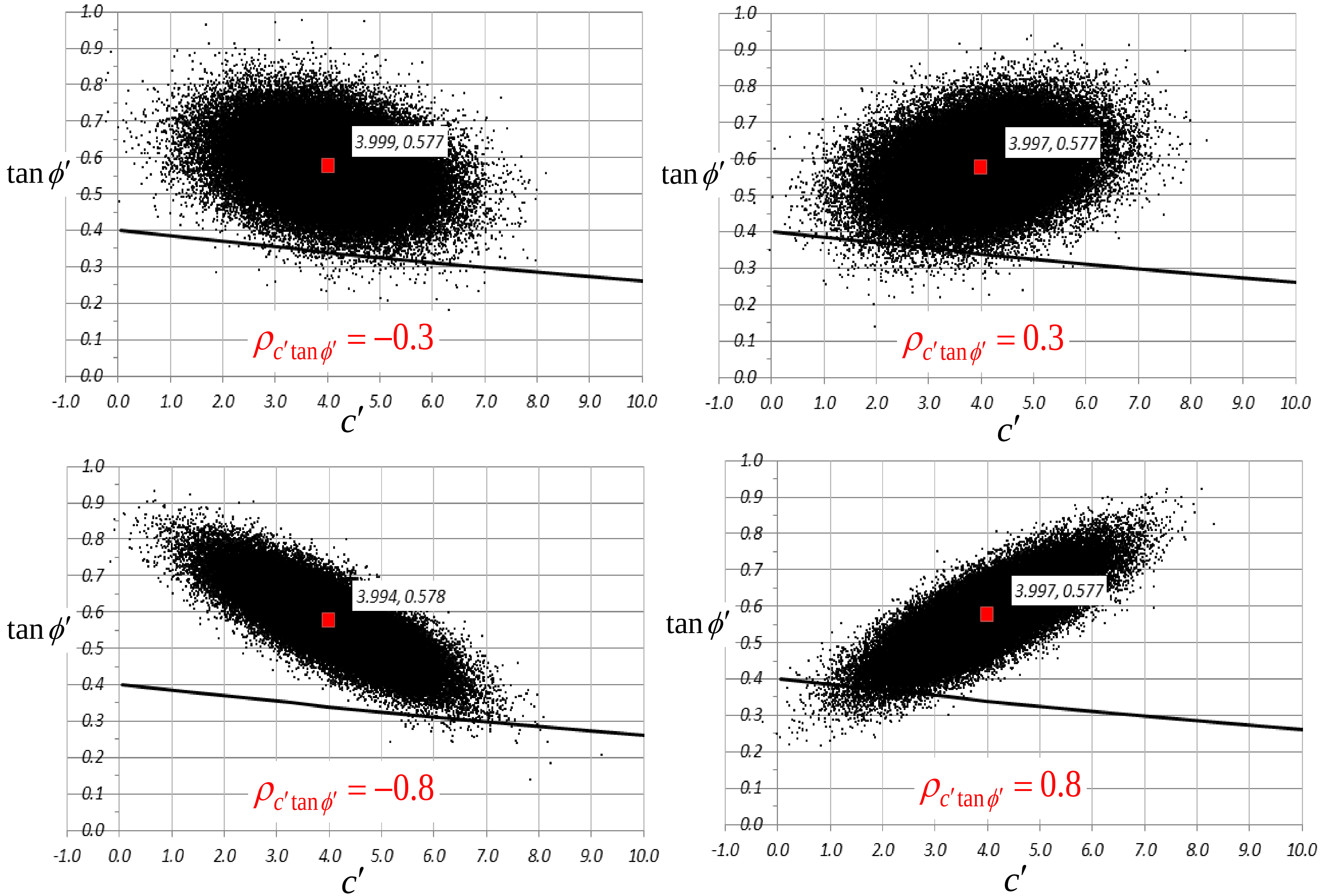
$\text{if}\left(q_{ult} < 300, 1, 0\right)$

99991	99989	4.0135	0.7169	35.64	36.06	48.91	53.14	1.74	1.2	1.72	1.12	0.6	1	2236.8	0
99992	99990	3.8673	0.5013	26.62	12.68	23.29	13.71	1.54	1.2	1.50	1.15	0.6	1	709.8	0
99993	99991	5.3660	0.6025	31.07	20.79	32.85	26.26	1.63	1.2	1.60	1.14	0.6	1	1313.5	0
99994	99992	2.4980	0.5942	30.72	19.98	31.94	24.93	1.63	1.2	1.59	1.14	0.6	1	1079.7	0
99995	99993	4.3333	0.5011	26.61	12.66	23.27	13.69	1.54	1.2	1.50	1.15	0.6	1	729.0	0
99996	99994	3.4208	0.5844	30.30	19.04	30.87	23.42	1.62	1.2	1.58	1.14	0.6	1	1078.7	0
99997	99995	1.3321	0.6016	31.03	20.70	32.75	26.11	1.63	1.2	1.60	1.14	0.6	1	1048.5	0
99998	99996	5.0243	0.6439	32.78	25.41	37.90	34.00	1.67	1.2	1.64	1.14	0.6	1	1602.5	0
99999	99997	3.6576	0.4219	22.87	8.55	17.89	8.06	1.48	1.2	1.42	1.16	0.6	1	456.4	0
100000	99998	4.6878	0.6000	30.96	20.54	32.57	25.85	1.63	1.2	1.60	1.14	0.6	1	1253.2	0
100001	99999	3.7898	0.6745	34.00	29.44	42.17	41.07	1.70	1.2	1.67	1.13	0.6	1	1772.9	0
100002	100000	3.4187	0.5693	29.65	17.69	29.32	21.28	1.60	1.2	1.57	1.15	0.6	1	995.0	0
100003	Mean	3.9994	0.5773											1180.6	212
100004	SD	0.9979	0.0861											547.5	
100005															
100006		c'	tanϕ'	ϕ'	N_q	N_c	N_γ	F_{cs}	F_{cd}	F_{qs}	F_{qd}	$F_{\gamma s}$	$F_{\gamma d}$	q_{ult}	p_f
100007		4	0.577	0.5233	18.37	30.10	22.35	1.61	1.2	1.58	1.14	0.6	1	1070.8	0.21%

n_{tot}

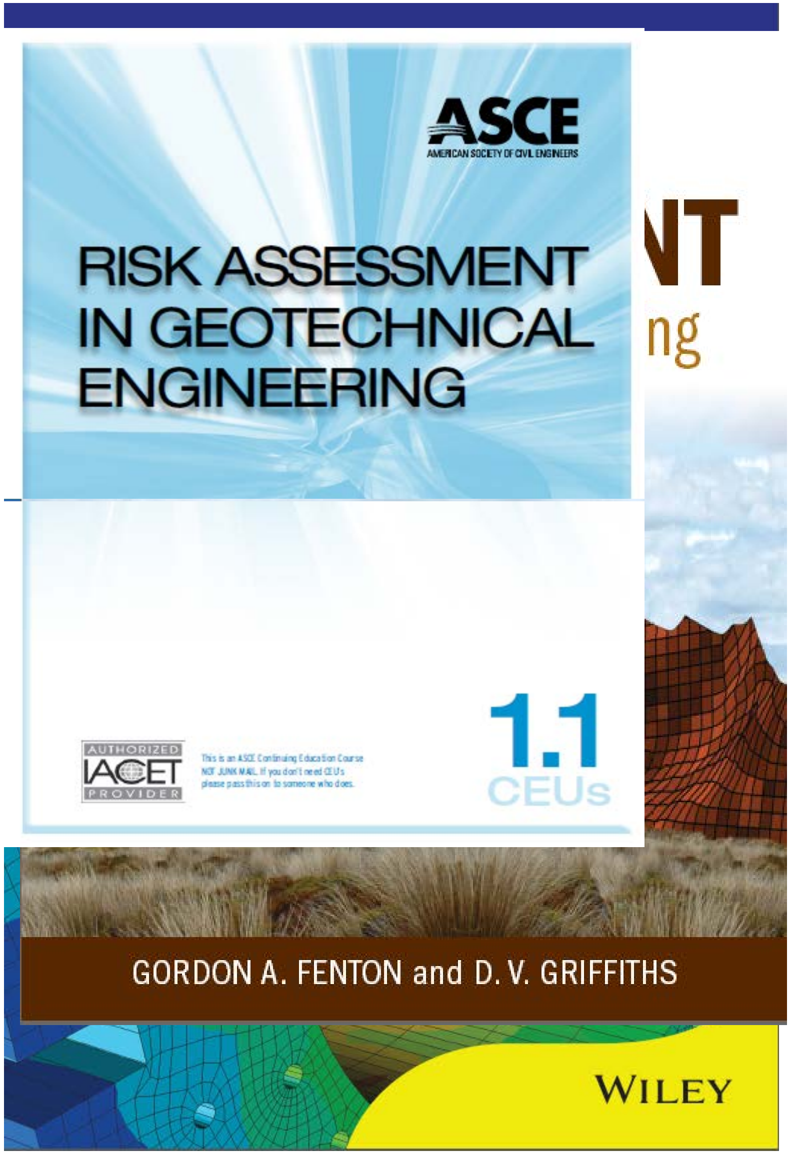
n_f

$p_f = n_f / n_{tot}$



The more positive the correlation between c' and $\tan\phi'$, the higher the p_f

Level 3: The Random Finite Element Method (RFEM)



Simulation of Random Fields via Local Average Subdivision

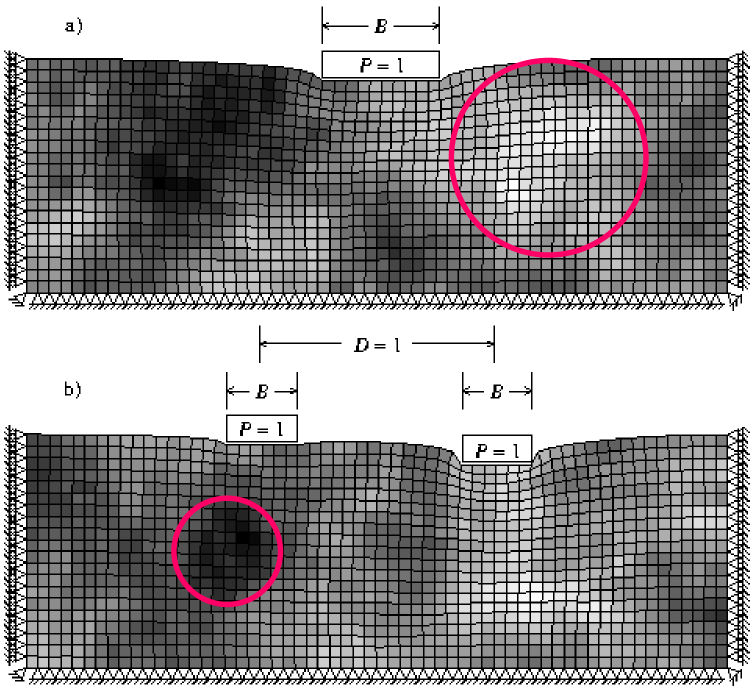
By Gordon A. Fenton[1] and Erik H. Vanmarcke,[2] Members, ASCE

This paper is part of the *Journal of Engineering Mechanics*, Vol. 116, No. 8, August, 1990. ©ASCE, ISSN 0733-9399/90/0008-1733/$1.00 + $.15 per page. Paper No. 24927.

- **Developed in the 1990s for advanced probabilistic geotechnical analysis.**
- **Combines finite element and random field methodologies in a Monte-Carlo framework.**
- **Properly accounts for (anisotropic) spatial correlation structures in soil deposits.**
- **Properly accounts for element size through local averaging.**
- **All programs are open-source.**
- **Frequent short courses given for ASCE and internationally**
- **Now a considerable bibliography on the method and included in proprietary codes.**

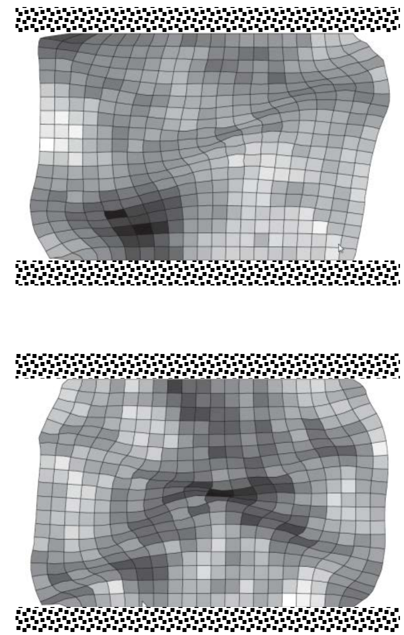
Geotechnical Applications

Settlement

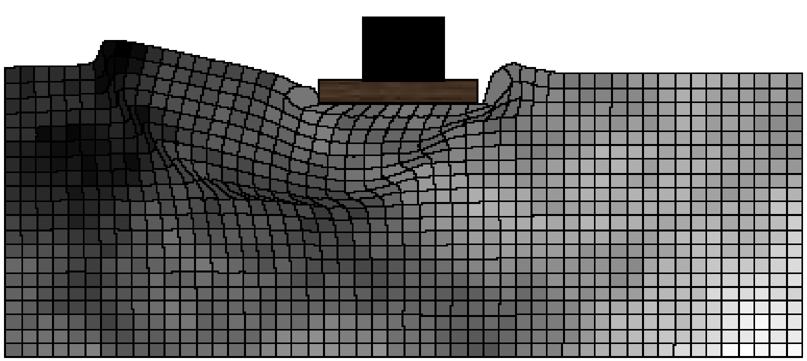


spatial
correlation
length

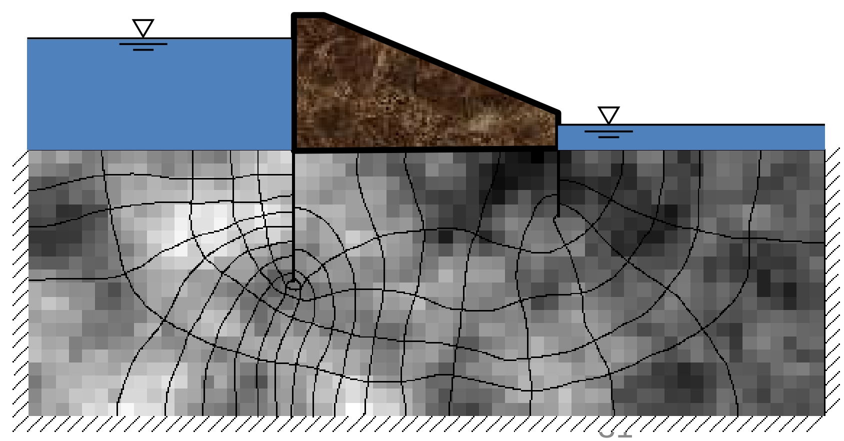
Mine pillar Stability



Bearing Capacity

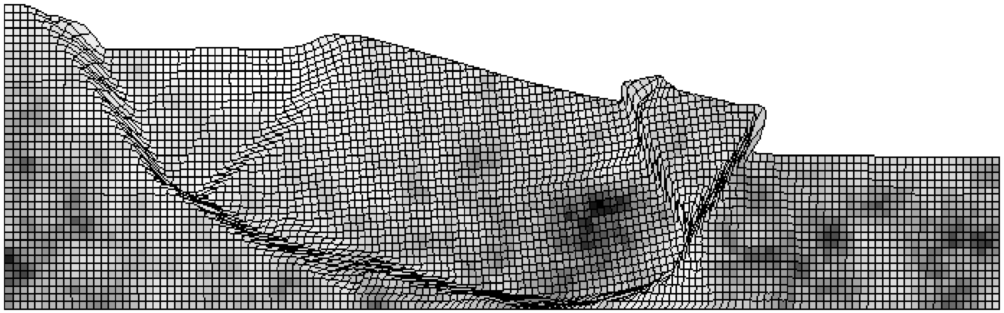


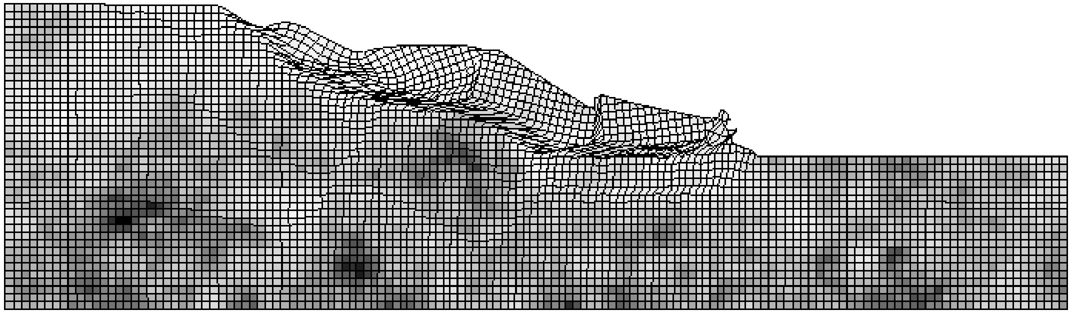
Seepage

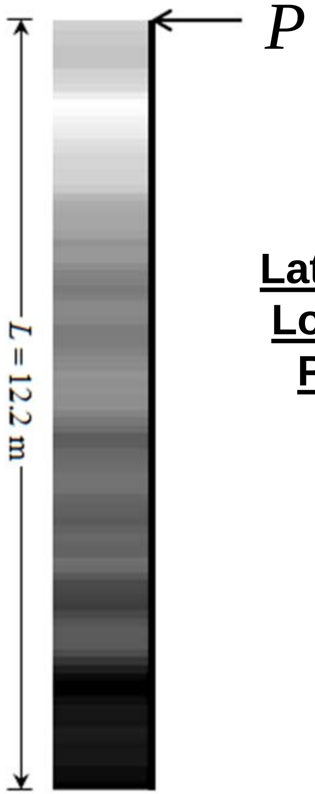


Earth Pressures

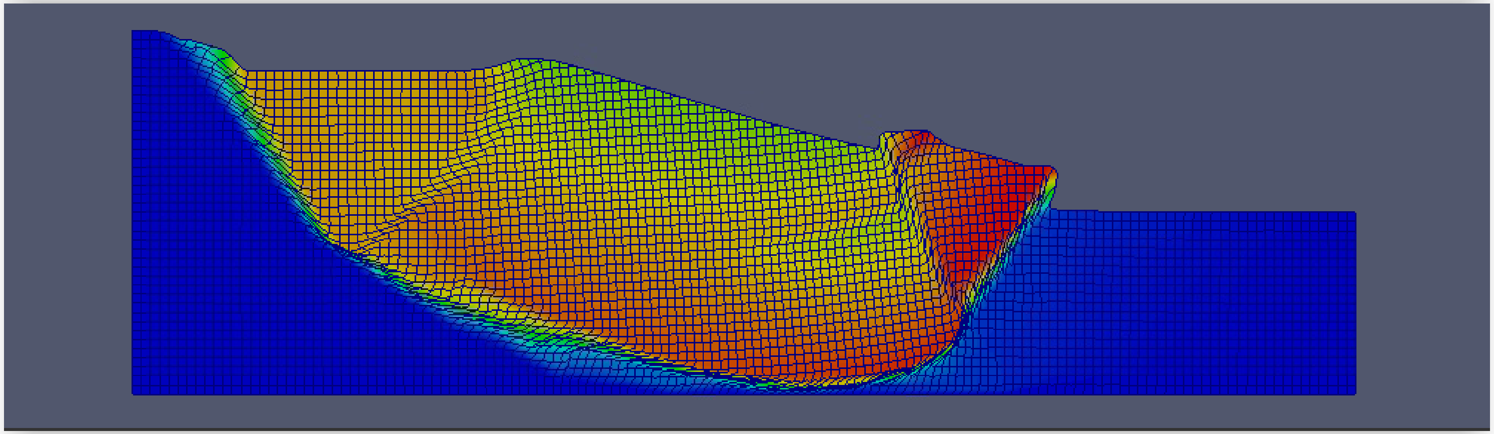
Slope Stability

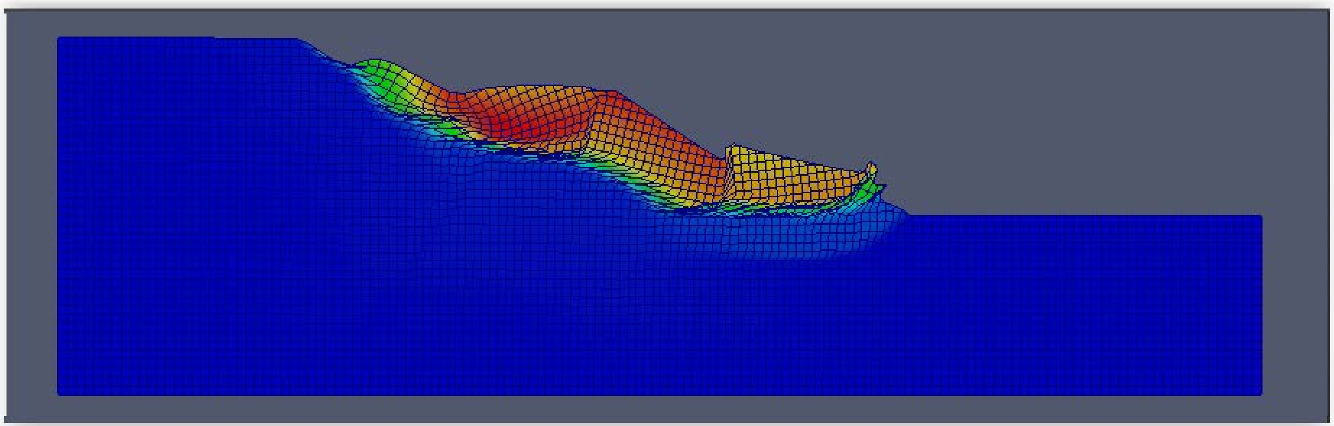






Laterally Loaded Piles





3) Concluding Remarks

- The natural variability of geomaterials makes them naturally suited to analysis using statistical methods

- Numerical discretization methods remain the most powerful methods for modeling variable soils. In stability analysis, FE “seeks out” the critical failure mechanism which is essential when dealing with random soils.

- Direct comparison between *FS* and p_f should be done with great care.

- For probabilistic geotechnical analysis, engineers have a toolbox of methods. Three levels of complexity have been identified, but only RFEM properly accounts for spatial variability.

All the programs described in this seminar can be downloaded from
www.mines.edu/~vgriffit

THANK YOU.